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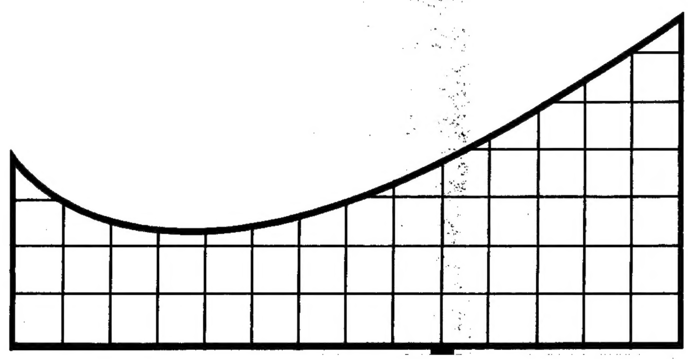
CALCULATING CREEP AND STRESS RELAXATION FROM LONG TERM DATA



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CALCULATING CREEP AND STRESS RELAXATION FROM LONG TERM DATA

There are many equations that have been and can be derived for the calculation of stresses and strains in bodies of various geometric shape. The modulus of elasticity of the material can also be derived from many of these same relationships. These equations assume (in most cases) that the body is elastic and obeys Hooke's Law. Also, they can only be applied with validity within the limit of proportionality of stress to strain. Properly used, these equations greatly facilitate the work of the designer and engineer who is dealing with the short time or elastic properties and can be successfully applied to design of plastic parts when stresses of short duration are anticipated.

Many of the applications for plastics today involve long term loading, or stressing, of the parts rather than short term loadings. The design and engineering of parts for long term applications cannot realisticly be based on short term data because plastics, like all other materials, are subject to a phenomenon under stresses of long duration known as "creep", "cold flow" or "stress relaxation". This means that if the material

is free to act under stress its dimensions will gradually change resulting in creep or cold flow. On the other hand, if the part is originally and deliberately deformed to some predetermined shape and then restrained in that position so that total deformation will neither increase nor decrease, then a second phenomenon known as stress relaxation takes place. In this case, the initial stress built up in the piece as it tries to resist being deformed gradually decreases and if left in that position long enough would theoretically decrease to zero. If then removed from the deforming force, the part would retain completely the shape into which it had been forced.

Actually the two phenomena of creep and stress relaxation are very closely related, and the end result is identical in that permanent deformation of the part occurs. Because of their close relationship, a knowledge of the creep behavior of a plastic will enable one to accurately calculate the amount of stress relaxation that would occur in those applications where the part is initially deformed and then restrained in that position. The reverse is also true.

Various applications for plastics make use of materials in such a way that they are subject to either creep, stress relaxation, or a combination of both.

The type of material, the temperature, the magnitude of the applied load, and the length of time the part is under stress will determine the rate and extent of creep and stress relaxation. A knowledge of the rate and extent to which materials will creep or relax is necessary to be able to predict and design for the expected performance of the material.

When long term strength data on materials are available (stress relaxation and/or creep) these same equations can be used by the designer and the engineer to obtain a good approximation of how much creep or stress relaxation will take place in a given period of time. The long term creep or stress relaxation data must be substituted for the usual short term elastic data in the equations. For example, if lawnmower wheels or motor housings are to be designed, data obtained over a five year period should be used. If, on the other hand, a rocket or missile component is to be made of plastic, then the stresses will be applied for a very short period of time and the short term data should be used.

Several examples of some of the simpler forms of equations used to calculate stress, strain, and the modulus of elasticity are:

1. Equations for calculating simple stresses.

(a). Stress in tension.

$$S = \frac{P}{A}$$

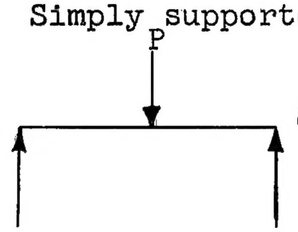
P = Applied load
A = Cross-sectional area of the part

(b). Stresses in Compression.

Same as for tension.

(c). Stress in flexure (rectangular beam).

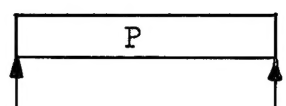
(1). Simply supported, center loaded beam.



$$S = \frac{3}{2}\,\frac{PL}{bd^2}$$

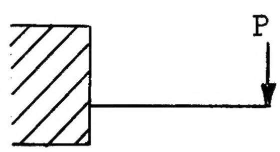
P = Applied load
L = Distance between supports
b = Width of beam
d = Depth of beam

(2). Simply supported, uniformly loaded beam.



$$S = \frac{3}{4}\,\frac{PL}{bd^2}$$

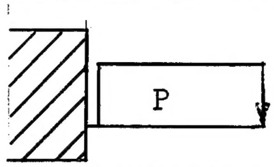
(3). Cantilever beam, end loaded.



$$S_{max.} = \frac{6\,PL}{bd^2}$$

P = Applied load
L = Distance from load to support
b = Width of beam
d = Beam depth

(4). Cantilever beam, uniformly loaded.



$$S_{max.} = \frac{3\,PL}{bd^2}$$

2. Equations for calculating modulus of elasticity in the preceding examples.

(a). Modulus of tension.

$$E = \frac{S}{\epsilon}$$

S = Stress
ϵ = Strain

(b). Modulus in compression.

Same as (a)

(c). Modulus in flexure.

(1). Simply supported, center loaded beam.

$$E = \frac{L^3 m}{4\ bd^3} \qquad m = \text{Initial slope of flexure curve}$$

(2). Simply supported beam, uniform load.

$$E = \frac{5}{32} \quad \frac{L^3 m}{bd^3}$$

(3). Cantilever beam, end loaded.

$$E = \frac{4\ L^3 m}{bd^3}$$

(4). Cantilever beam, uniform load.

$$E = {}^3/_2 \quad \frac{L^3 m}{bd^3}$$

Using these equations of state and the long term data such as that given in Figures I, II, III, IV, and V, the creep or stress relaxation occurring in a plastic part held under stress can be calculated. The following examples and problems demonstrate the application of long term data. The long term data used in these examples were obtained on Tyril 767, Styron 475, and Dow saran monofilament.

Example 1

From the expression of modulus of elasticity in tension and the curve of "apparent modulus" vs. time, the creep or stress relaxation can be calculated if a limit can be determined for one or the other. Let's assume that for Tyril 767 the tensile stress to be applied is 1,000 psi, and we want to know how much creep will occur in a part during the first six months (4,320 hrs.). From Figure II the apparent modulus at six months is 420,000 psi.

Equation 2(a) $E = \frac{S}{\epsilon}$ and $\epsilon = \frac{S}{E}$

$$\epsilon = \frac{1,000}{420,000} = 0.0024 \text{ inches/inch or } 0.24\% \text{ elongation}$$

Example 2

If, on the other hand, the application is such that a known deformation is to be imposed on the part (example: plastic closure for a bottle) then the amount of stress (resistance) still remaining in the part after a given time may be desired. Assuming the strain to be imposed will be 0.42%, the stress still remaining after one year is found as follows: (apparent modulus @ 1 year = 380,000 psi)

Equation 2(a) $S = E\epsilon = (380,000)\ (0.0042) = \underline{\underline{1500}}$ psi

Problem No. 1

Plastic monofilament is quite commonly used to manufacture the webbing or straps used in making office, home and lawn furniture. The filaments are usually circular in cross section with a diameter of about 10 mil and are highly oriented. In an ordinary chair, there will be five straps across the seat and five straps parallel to the seat for supporting the weight of the person sitting in the chair.

Designers, of course, wish to use the most economical material and this is dependent on the following factors:

1. Cost per pound of base resin.
2. Density of base resin.
3. Fabrication and weaving costs.
4. The design strength of the material.

Once the correct material has been chosen, the designer must then determine the number of strands to be used in each of the straps. This he can determine from the design strength of the material and the size of the monofilament to be used.

In this hypothetical problem, the designer has two satisfactory materials to choose from and he must decide which is more economical and how many strands will be required in each web or strap. His design is to be based

on a five year expected service life and the webbing must not develop a permanent sag greater than one inch at the center during the five year period. The design will also assume ambient temperatures of 75 to 80°F, and the size of the chair seat is to be 19 by 19 inches.

The following data on the two materials have been supplied to the designer:

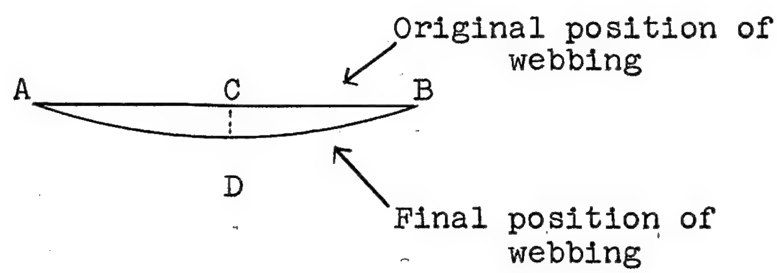
		Material A	Material B
1.	Design strength, psi	See curve of apparent modulus vs. time attched.	
2.	Safety factor	-------------2-----------	
3.	Specific gravity	0.995	1.12
4.	Cost per pound of base resin	\$0.54	\$0.42
5.	Fabrication cost	----------equal----------	
6.	Weaving cost	----------equal----------	

Solution

As previously stated the chair is to be designed such that the load as a person sits in the chair will be uniformly distributed over ten straps. The maximum load to be encountered is assumed to be 300 pounds and with a safety factor of two, the working load then becomes 600 pounds and each strap must then support 60 pounds. The straps are fastened at each end, so the load at each end will then be 30 pounds. It is assumed also that the

chair will be occupied approximately 6 hours per day or one and one-quarter years over a period of five years.

The first step in the solution to the problem is to determine how much creep can be permitted to take place in the straps before the design limit of the one inch permanent deflection at the center of the seat is reached. The sag or permanent deformation taking place in the webbing can be treated as a segment of the arc of a circle and the strain or creep taking place in the monofilaments is calculated in the following manner:



Cross-Section of Chair Seat

AB = Original length of webbing = 19 inches.

CD = Maximum allowable permanent deflection (5 Yrs.) = 1 inch.

ADB = Final length of webbing and is the arc of a circle subtended by chord AB.

Then ADB = $\frac{\pi r \theta}{180}$

Where: r = Radius of the circle = $1/2 \frac{AC^2 + CD^2}{CD}$

θ = Angle included by the radii connecting points

A and B = Arcsin $\frac{AB}{r}$

$$ABD = \frac{\pi \left(1/2 \; \frac{AC^2 + CD^2}{CD}\right) \left(\text{Arcsin} \; \frac{AB}{1/2 \; \frac{AC^2 + CD^2}{CD}}\right)}{180}$$

$$= \frac{(3.14) \left(1/2 \; \frac{(9.5)^2 + (1)^2}{1}\right) \left(\text{Arcsin} \; \frac{19}{1/2 \; \frac{(9.5)^2 + (1)^2}{1}}\right)}{180}$$

$$= 19.6 \text{ inches}$$

$$\text{Stress (creep)} = \frac{\text{Change in length}}{\text{Original length}} = \frac{0.6}{19} = 0.032 \; \frac{\text{in.}}{\text{in.}}$$

Next it is necessary to calculate the limiting stress for each of the two materials. Since plastics will undergo permanent deformation in the form of creep when a load is applied, the limiting stress must be calculated from creep data obtained in a series of experiments on these two materials. For convenience to the designer, these data from numerous experiments are represented by a curve of apparent modulus versus time. The simple relationship, apparent modulus, psi, equal to stress, psi divided by strain, in./in., is used to calculate the limiting stress when the allowable creep is known. This is done as follows:

From Figure III the apparent moduli for the two materials is obtained.

	Material A	Material B
Apparent modulus, psi (initial)	2.42×10^5	1.85×10^5
Apparent modulus, psi (1-1/4 years)	1.45×10^5	0.88×10^5

Design stress for Material A

$$S_D = (E)\ (\epsilon) = (E_2)\ (\epsilon_1 + \epsilon_2)$$

Where: S_D = Design stress

E_2 = Apparent modulus (1-1/4 yrs.) = 1.45×10^5 psi

ϵ = Total strain on the webbing = $\epsilon_1 + \epsilon_2$

ϵ_1 = Initial elastic strain = $\frac{S_D}{E_2}$

ϵ_2 = Permanent deformation (creep) = 0.32 $\frac{\text{inches}}{\text{inch}}$

E_1 = Apparent modulus (initial) = 2.42×10^5 psi

$$S_D = (E_2)\ \left(\frac{S_D}{E_1} + \epsilon_2\right) = (1.45 \times 10^5)\ \left(\frac{S_D}{2.42 \times 10^5} + 0.032\right)$$

$$= 0.6\ S_D + 4640$$

$$= \frac{4640}{0.4} = \underline{\underline{11,600}} \text{ psi}$$

Design stress for Material B

$$S_D = (0.88 \times 10^5)\ \left(\frac{S_D}{1.85 \times 10^5} + 0.032\right)$$

$$= \underline{5,350} \text{ psi}$$

The designed is now is a position to determine which of the two materials is more economical. Since the fabrication and weaving costs for the two materials are essentially the same, the only factors remaining to be considered are the

cost per pound, the specific gravity and the design strength. For purposes of comparison, these factors can be related in an equation to obtain the cost index for each material.

$$\text{Cost Index} = \frac{(\text{Resin cost/pound})\ (\text{Specific gravity})}{\text{Design strength}}$$

$$\text{Cost Index (Material A)} = \frac{(0.54)\ (0.995)}{11{,}600} = 0.0032$$

$$\text{Cost Index (Material B)} = \frac{(0.42)\ (1.12)}{5{,}350} = 0.0057$$

From these results, it can be concluded that Material A is the most economical choice by a factor of almost two to one. The remaining problem which the designer must answer is to determine how many strands each web or strap must contain in order to support the anticipated loads.

$$\text{No. of strands required per strap} = \frac{A_{total}}{A_{one}} = \frac{\frac{P}{S_D}}{A_{one}}$$

Where: A_{total} = Total cross section of material required.

A_{one} = Cross sectional area of one monofilament.

P = Load applied at the end of each strap = 30 lbs.

S_D = Design stress.

$$\text{Strands/strap} = \frac{\frac{P}{S_D}}{A_{one}} = \frac{\frac{P}{S_D}}{\pi r^2}$$

$$= \frac{\frac{30}{11{,}600}}{(3.14)(0.005)^2} = \underline{\underline{33}}$$

Thirty-three monofilaments in a webbing would be rather narrow so the designer can either reduce the size of each monofilament to gain a larger total number, and hence a wider strap or he can specify additional strands to be added even though they are not necessary.

Problem 2

A filter sump must be designed for use in a home water system. The filter must have a life expectancy of five years and operate at a maximum pressure of 75 psi internal pressure. The design also dictates that, to insure a sufficient O-ring seal, the total circumferential deformation of the sump cannot exceed a change in radius of 0.007 in. If this amount of creep is exceeded, the sump will leak.

Solution

The filter sump is a thick-wall cylinder, the limiting stress would be the hoop stress, and the following equations would apply for calculating fiber stress and change in radius:

$$S = \text{Hoop Stress} = P \frac{a^2 (b^2 + r^2)}{r^2 (b^2 - a^2)}$$

$$\Delta a = P \frac{a}{E} \left[\frac{b^2 + a^2}{b^2 - a^2} - v \left(\frac{a^2}{b^2 - a^2} \right) - 1 \right]$$

Where:

a = Inner radius = 2.05 in.

b = Outer radius = 2.30 in.

r = Nominal radius = 2.175 in.

v = Poisson's Ratio = 0.305

Δa = Change of inner radius, in.

S = Hoop stress, psi

P = Internal pressure, psi

The material proposed for this application, because of its good long term strength (Figure I and II), ease of fabrication, and low cost was Tyril 767. The design strength in a creep application such as this, obtained from an extrapolation of the stress relaxation curve (Figure I), is 980 psi. The maximum allowable operating pressure can then be calculated from the equation:

$$S = P \frac{a^2 (b^2 + r^2)}{r^2 (b^2 - a^2)}$$

Solving for P:

$$P = \frac{S \left[r^2 (b^2 - a^2)\right]}{a^2 (b^2 + r^2)}$$

$$= \frac{980 \left[4.73 (1.09)\right]}{4.2 (10.02)} = \underline{\underline{120}} \text{ psi}$$

The maximum allowable pressure found here is well above the operational requirements. The designer must then determine if the change in radius due to internal pressure is within the design limitations to insure that the O-ring will maintain an adequate seal.

$$\Delta a = P\ \frac{a}{E}\left[\frac{b^2 + a^2}{b^2 - a^2} - \left(v\ \frac{a^2}{b^2 - a^2} - 1\right)\right]$$

$$= \frac{75\ (2.05)}{5.23 \times 10^5}\left[\frac{9.49}{1.09} - 0.305\left(\frac{4.2}{1.09} - 1\right)\right]$$

$$= 0.00225 \text{ in.}$$

The five year design strength of Tyril 767 therefore affords a safety factor of 1.6 for the operating pressure and three for the O-ring seal. It is possible that a slight decrease in wall section thickness could be considered for economic reasons. However, the designer will probably want to maintain a safety factor of 1.5 against contingencies.

Problem 3

An engineer has designed an adjustable television receiver antenna which is to be mounted on the back of a portable television set. The antenna mounting well houses a spring which is compressed and inserted between the antenna rod and a spring washer. The spring washer rests against a ledge on the inside of the mounting well.

The spring is compressed 9/16" and exerts a force of 40 lbs. on the spring washer. The cross sectional area of the mounting well wall supporting the spring washer is 0.230 square inches.

A high impact polystyrene is being considered for this application and the designer must know whether the part will last ten years under the present design loading or if the material will creep enough to allow the antenna rod to become loose in the socket making it unserviceable. The spring must maintain a force of 35 lbs. on the base of the antenna rod to keep it operable.

Solution

The antenna housing will be mounted on the back of the cabinet where the temperature will be approximately 100°F while the set is in operation. Since viscoelastic materials stress relax and creep at a faster rate at elevated temperatures, the designer uses a safety factor of 2.0 to compensate for the times when the set is turned on.

The stress on the well wall is:

$$S = \frac{P\ (S_f)}{A}$$

Where: P = Load, lbs. = 40 lbs.

S_f = Safety factor = 2.0

A = Cross sectional area = 0.230 in.^{2}

$$\text{Stress} = \frac{40\ (2)}{.230} = 350 \text{ psi}$$

The apparent modulus and stress relaxation data, extrapolated to ten years, for Styron 475 are shown in

Figures IV and V respectively. The stress remaining in the well wall after ten years is calculated as follows:

$$\epsilon = \frac{S_o}{E_o}$$

$$S_{10} = (\epsilon)\ (E_{10})$$

Where: S_o = Initial stress = 350 psi

E_o = Modulus of elasticity = 3.8×10^5 psi

S_{10} = Stress remaining @ 10 years.

E_{10} = 10 years apparent modulus

= 1.4×10^5 psi (Figure IV)

ϵ = Strain

By calculation:

$$\epsilon = \frac{350}{3.8 \times 10^5} = \begin{array}{l}0.00092 \text{ inches/inch} \\ 0.092\% \text{ strain}\end{array}$$

The remaining stress is:

$$S_{10} = (0.00092)\ (1.4 \times 10^5 \text{ psi}) = 130 \text{ psi}$$

The ten years design stress of Styron 475 is 220 psi (Figure V) which is high enough over the 130 psi remaining stress to give an additional degree of safety.

To determine whether the part will creep enough to allow the antenna rod to become loose the designer must assume the load to be constant, thereby imposing a constant stress of 350 psi. The following formulas apply:

$$\epsilon_{10} = \frac{S_O}{E_{10}}$$

$$P_{10} = L_{10}\ (K)$$

Where: $L_{10} = L_0 - \epsilon_{10}$

ϵ_{10} = Creep

S_0 = Initial stress = 350 psi

E_{10} = Apparent modulus @ 10 years = 1.40×10^5 psi

P_{10} = Spring force @ 10 years

L_0 = Original amount spring is compressed = 0.5625 inches

$L_{10} = L_0 - \epsilon_{10}$ = Amount spring is compressed after creep

By calculation: K = Spring constant at = 71.2 lbs./inch

$$\text{Creep} = \epsilon_{10} = \frac{350}{1.4 \times 10^5} + 2.5 \times 10^{-3} = 0.0025$$

$$L_{10} = .5625 - .0025 = 0.56$$

Therefore, the spring force is:

$$P_{10} = L_0\ (K) = (0.56)\ (71.2)$$

$$= 39.8 \text{ lbs.}$$

The amount of creep occurring in ten years, at this stress level is insignificant. The part in its present design will perform satisfactorily with Styron 475.

These examples and problems have been relatively simple situations. Many applications also involve temperatures and environments other than those shown here and some may involve "shock" or rapid stressing of the part. The effect of such additional factors should be taken into consideration.

STRESS RELAXATION OF COMPRESSION MOLDED TYRIL 767 AT 73° F.

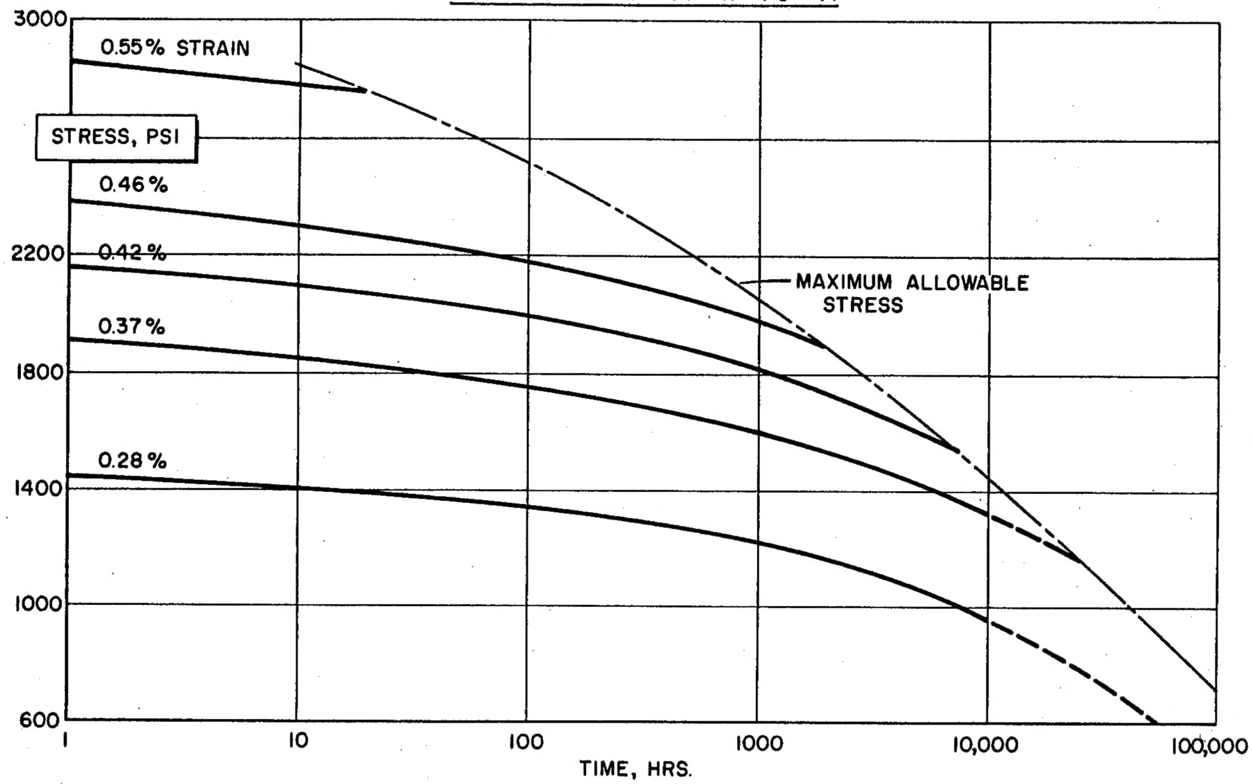


FIGURE I

APPARENT MODULUS OF COMPRESSION MOLDED TYRIL 767 AT 73° F.

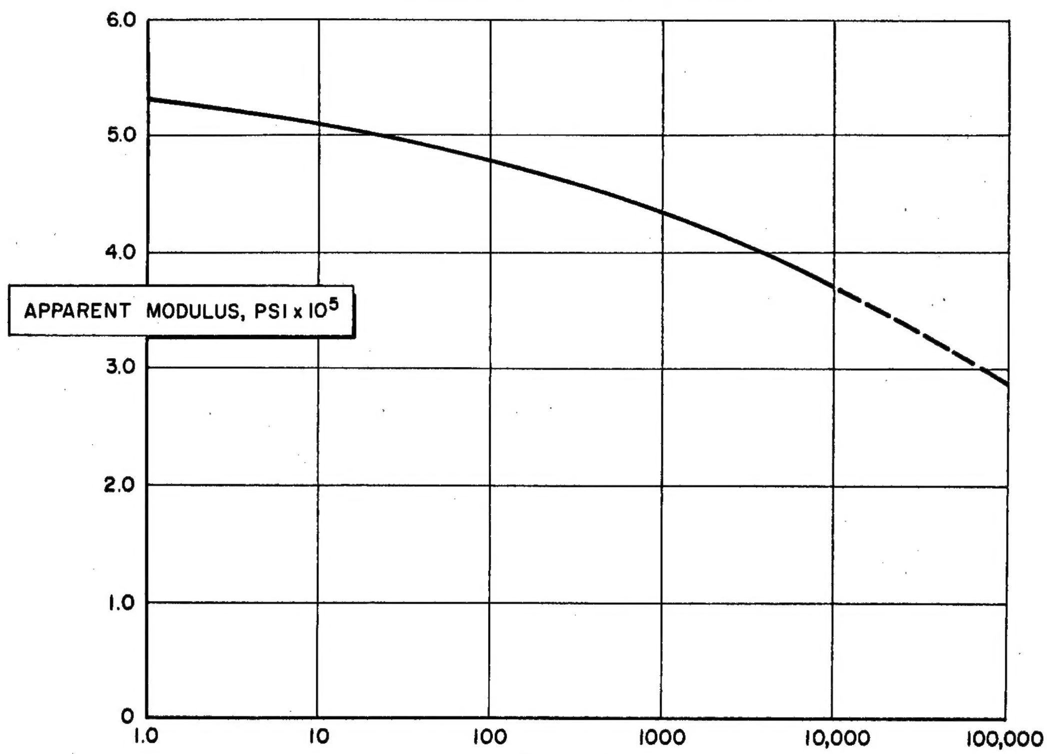


FIGURE II

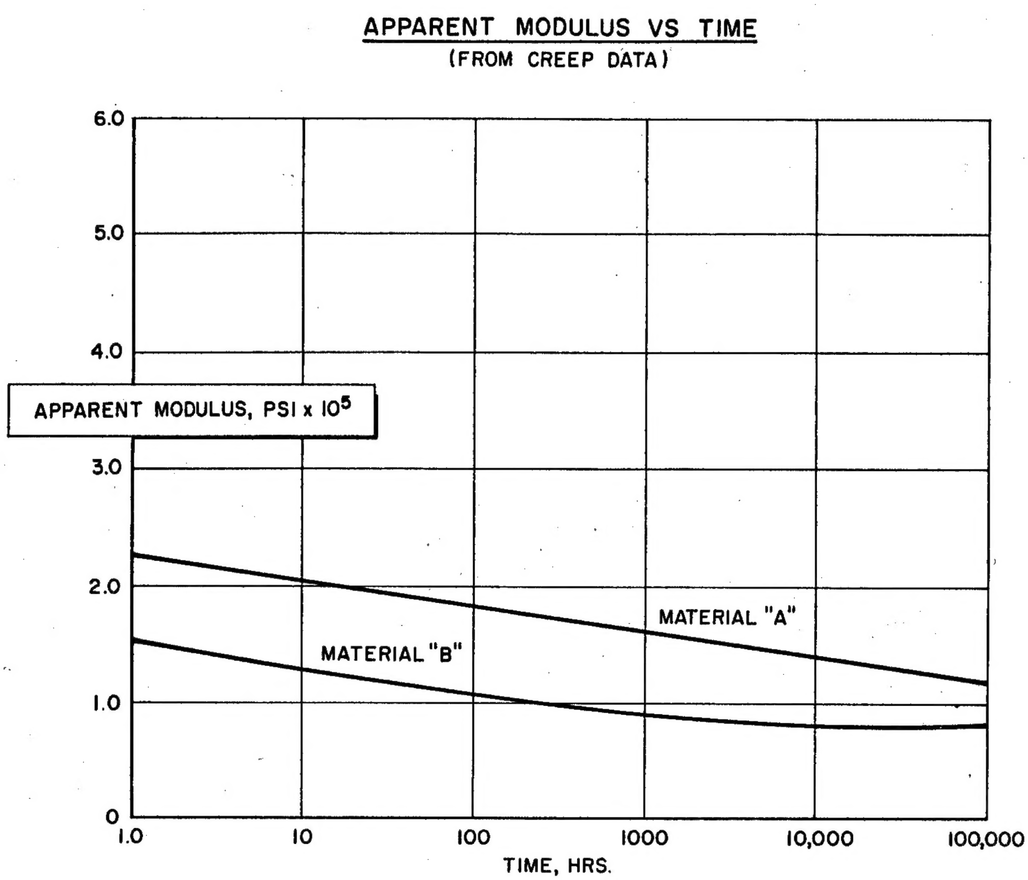


FIGURE III

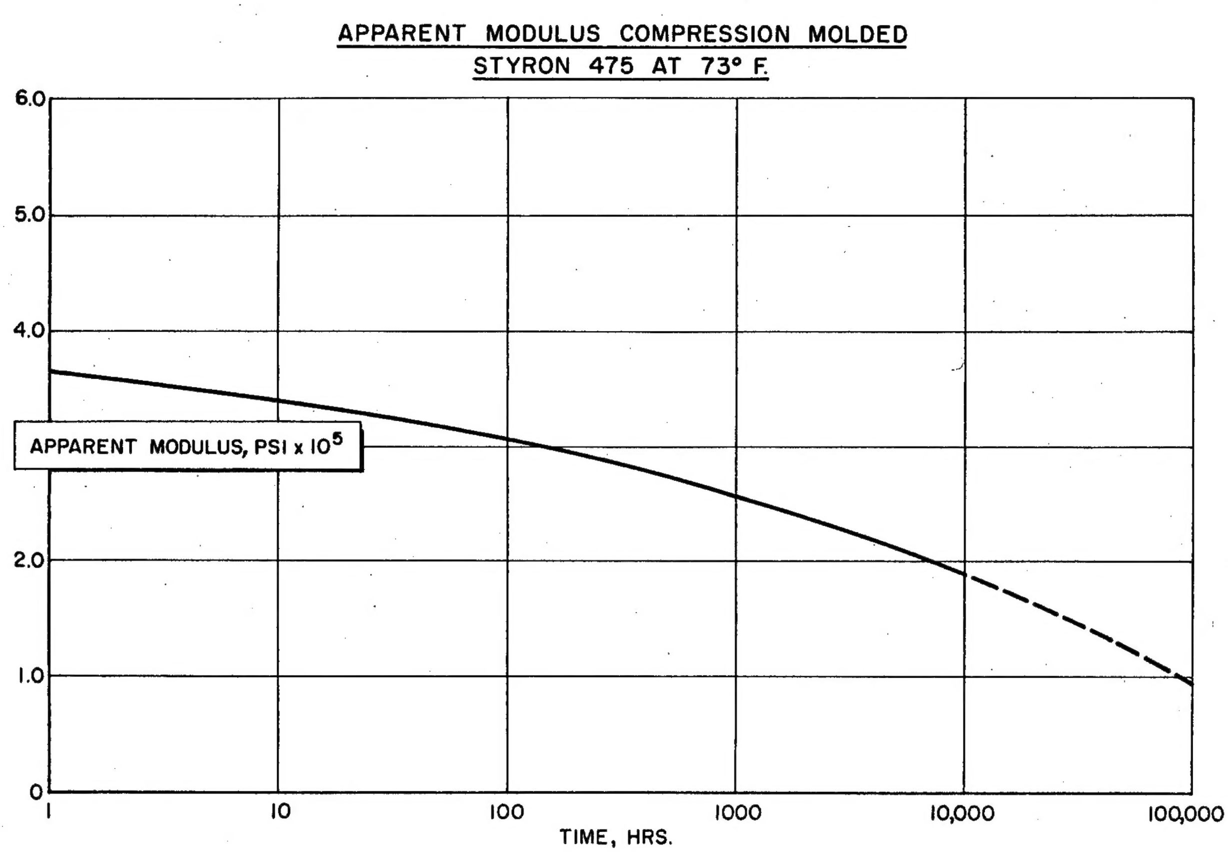


FIGURE IV

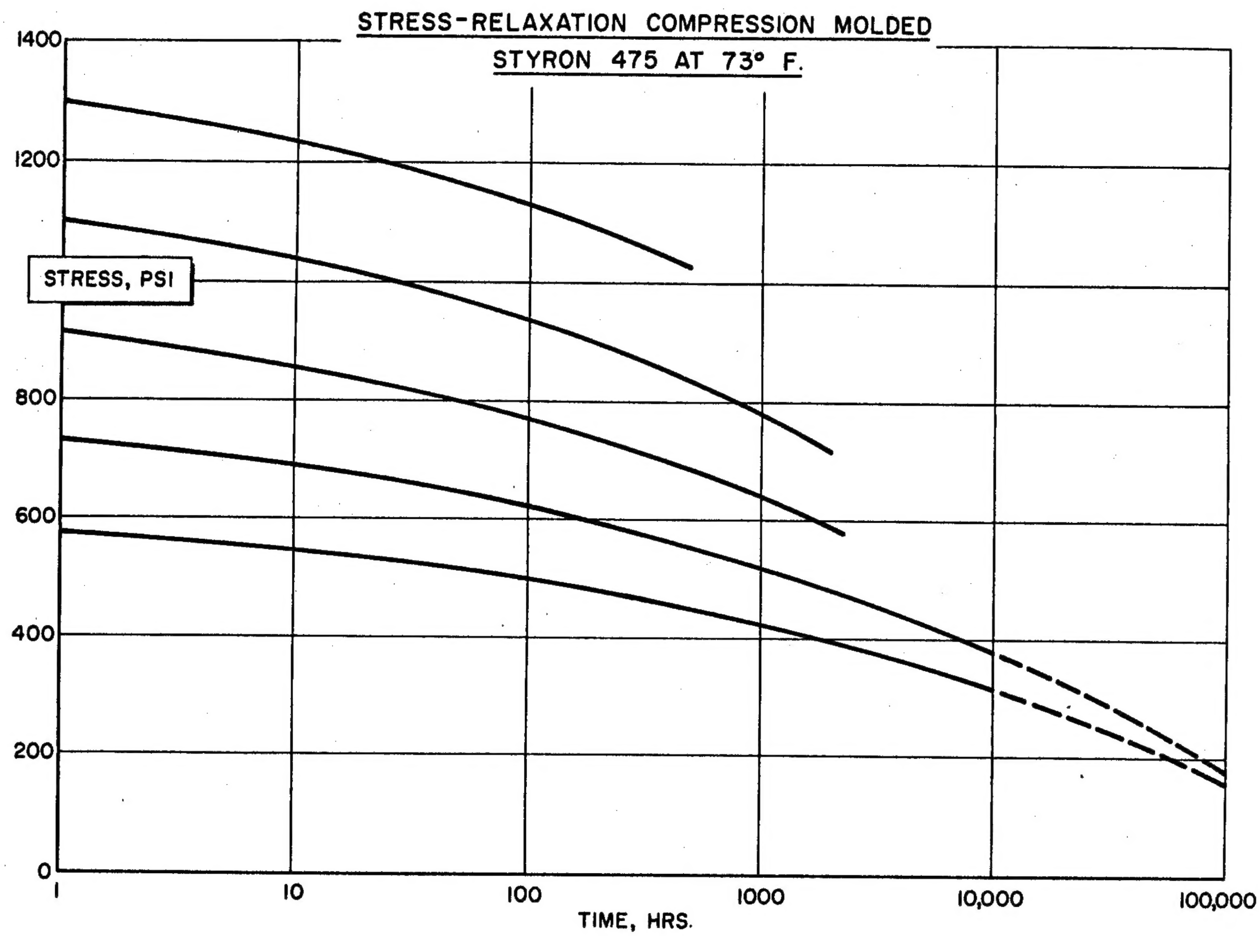

STRESS-RELAXATION COMPRESSION MOLDED
STYRON 475 AT 73° F.
STRESS, PSI
1400
1200
800
600
400
200
0
1
10
100
1000
10,000
100,000
TIME, HRS.

FIGURE V